

Predictions for Neutrinoless Double-Beta Decay in the 3+1 Sterile Neutrino Scenario

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We present accurate predictions of the effective Majorana mass $|m_{\beta\beta}|$ in neutrinoless double- β decay in the standard case of 3ν mixing and in the case of 3+1 neutrino mixing indicated by the reactor, Gallium and LSND anomalies. We have taken into account the uncertainties of the neutrino mixing parameters determined by oscillation experiments. It is shown that the predictions for $|m_{\beta\beta}|$ in the cases of 3ν and 3+1 mixing are quite different, in agreement with previous discussions in the literature, and that future measurements of neutrinoless double- β decay and of the effective light neutrino mass in β decay or the total mass of the three lightest neutrinos in cosmological experiments may distinguish the 3ν and 3+1 cases if the mass ordering is determined by oscillation experiments. We also present a relatively simple method to determine the minimum value of $|m_{\beta\beta}|$ in the general case of N -neutrino mixing.

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I. INTRODUCTION

Neutrino flavor oscillations have been observed in many solar, reactor and accelerator experiments (see the recent reviews in Refs. [1, 2]), in agreement with the currently standard paradigm of three-neutrino (3ν) mixing. The global fits of neutrino oscillation data in the framework of 3ν mixing [3–5] give us rather precise information on the values of the elements of the three mixing angles which parameterize the neutrino mixing matrix and on the values of the two independent neutrino squared-mass differences, the smaller “solar” squared-mass difference $\Delta m_{\text{SOL}}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2$ and the larger “atmospheric” squared-mass difference $\Delta m_{\text{ATM}}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2$.

However, the standard 3ν mixing paradigm has been challenged by indications in favor of short-baseline oscillations generated by a new larger squared-mass difference $\Delta m_{\text{SBL}}^2 \sim 1 \text{ eV}^2$: the reactor antineutrino anomaly [6], which is a deficit of the rate of $\bar{\nu}_e$ observed in several short-baseline reactor neutrino experiments in comparison with that expected from the latest calculation of the reactor neutrino fluxes [7, 8]; the Gallium neutrino anomaly [9–13], consisting in a short-baseline disappearance of ν_e measured in the Gallium radioactive source experiments GALLEX [14] and SAGE [15]; the signal of short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations observed in the LSND experiment [16, 17]. The simplest extension of 3ν mixing which can describe these short-baseline oscillations taking into account other constraints is the 3+1 mixing scheme [18, 19], in which there is an additional massive neutrino at the eV scale and the masses of the three standard neutrinos are much smaller. Since from the LEP measurement of the invisible width of the Z boson we know that there are only three active neutrinos (see Ref. [20]), in the flavor basis the additional massive neutrino corresponds to a sterile neutrino [21], which does not have standard weak interactions.

A fundamental question that remains open is: are neutrinos Dirac or Majorana particles? This question cannot be investigated in neutrino oscillation experiments, where the total lepton number is conserved and there is no difference between Dirac neutrinos with a conserved total lepton number and truly neutral Majorana neutrinos, which do not have a conserved total lepton number. The most promising process which can reveal the Majorana nature of neutrinos is neutrinoless double-beta decay, in which the total lepton number changes by two units (see the recent review in Ref. [22]).

From the present knowledge of the neutrino squared-mass differences and mixing angles it is possible to predict the possible range of values for the effective Majorana mass $|m_{\beta\beta}|$ in neutrinoless double-beta decay as a function of the absolute scale of neutrino masses (see Ref. [22]), which is still unknown, up to the upper bound of about 2 eV at 95% C.L. established by the Mainz [23] and Troitsk [24] Tritium β -decay experiments.

The introduction of a sterile neutrino at the eV mass scale can change dramatically the prediction for the possible range of values for the effective Majorana mass in neutrinoless double-beta decay [13, 25–33]. In this paper we present accurate predictions for $|m_{\beta\beta}|$ taking into account the results of the global fit of solar, atmospheric and long-baseline reactor and accelerator neutrino oscillation data presented in Ref. [3] and the results of an update [34, 35] of the global fit of short-baseline neutrino oscillation data presented in Ref. [19]. We are particularly interested to determine accurately the conditions for which $|m_{\beta\beta}| \gtrsim 0.01 \text{ eV}$, which may be probed experimentally in the near future (see Refs. [36–41]), and the conditions for which there can be a cancellation of the different mass contributions to $|m_{\beta\beta}|$, which leads to an unfortunate uncertainty for the possibility of ever observing neutrinoless double-beta decay (unless it is induced by new interactions and/or the exchange of new particles;

see Refs. [42–49]).

The plan of this paper is as follows. In Section II we discuss the predictions for $|m_{\beta\beta}|$ in the standard 3ν framework, taking into account the two possible normal and inverted mass orderings. In Section III we discuss how these predictions are modified in the $3+1$ mixing framework. In Section IV we draw our conclusions.

II. THREE-NEUTRINO MIXING

In the standard three-neutrino (3ν) mixing framework, the effective Majorana mass in neutrinoless double-beta decay is given by

$$|m_{\beta\beta}| = |\mu_1 + \mu_2 e^{i\alpha_2} + \mu_3 e^{i\alpha_3}|, \quad (1)$$

where

$$\mu_k = |U_{ek}|^2 m_k \quad (2)$$

is the partial contribution of the massive Majorana neutrino ν_k with mass m_k . The elements U_{ek} of the mixing matrix, which quantify the mixing of the electron neutrino with the three massive neutrinos, can have unknown complex phases, which generate the two complex phases α_2 and α_3 in Eq. (1). Since the values of these phases is completely unknown, all predictions of the value of $|m_{\beta\beta}|$ must take into account all the possible range of these phases, from 0 to 2π .

We use the results of the global fit of solar, atmospheric and long-baseline reactor and accelerator neutrino oscillation data presented in Ref. [3], which are given in terms of the mixing angles ϑ_{12} , ϑ_{13} that determine the absolute values of the first row of the mixing matrix U in the standard parameterization:

$$|U_{e1}| = \cos \vartheta_{13} \cos \vartheta_{12}, \quad (3)$$

$$|U_{e2}| = \cos \vartheta_{13} \sin \vartheta_{12}, \quad (4)$$

$$|U_{e3}| = \sin \vartheta_{13}. \quad (5)$$

The results for the neutrino squared-mass differences are expressed in terms of the solar and atmospheric squared mass differences, which are defined by

$$\Delta m_{\text{SOL}}^2 = \Delta m_{21}^2, \quad (6)$$

$$\Delta m_{\text{ATM}}^2 = \frac{1}{2} |\Delta m_{31}^2 + \Delta m_{32}^2|, \quad (7)$$

where $\Delta m_{jk}^2 = m_j^2 - m_k^2$. Given this assignment of the squared mass differences, it is currently unknown if the ordering of the neutrino masses is normal (NO), such that $m_1 < m_2 < m_3$ or inverted (IO), such that $m_3 < m_1 < m_2$. We discuss these two cases separately in the following subsections.

A. Normal Ordering

In order to study the case of Normal Ordering (NO), we express the neutrino masses in terms of the lightest

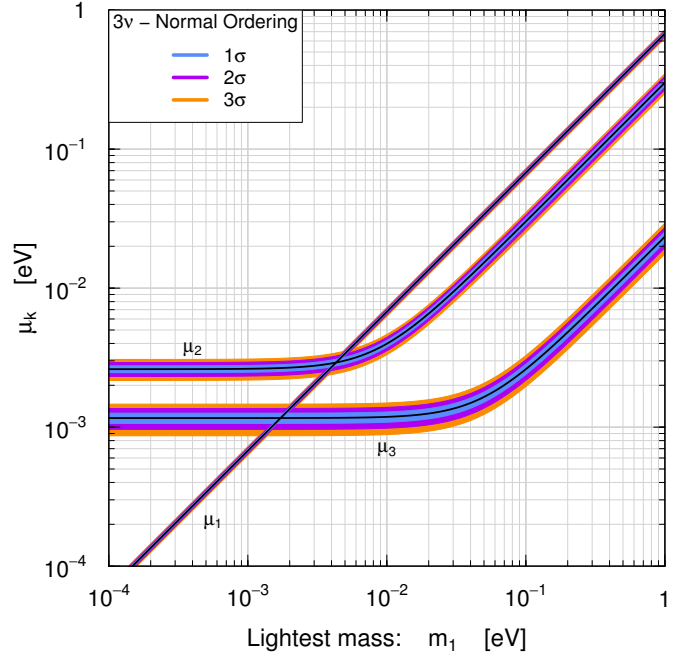


FIG. 1. Best-fit values (b.f.) and 1σ , 2σ and 3σ allowed intervals of the three partial mass contributions to $|m_{\beta\beta}|$ in Eq. (1) as functions of the lightest mass m_1 in the case of 3ν mixing with Normal Ordering.

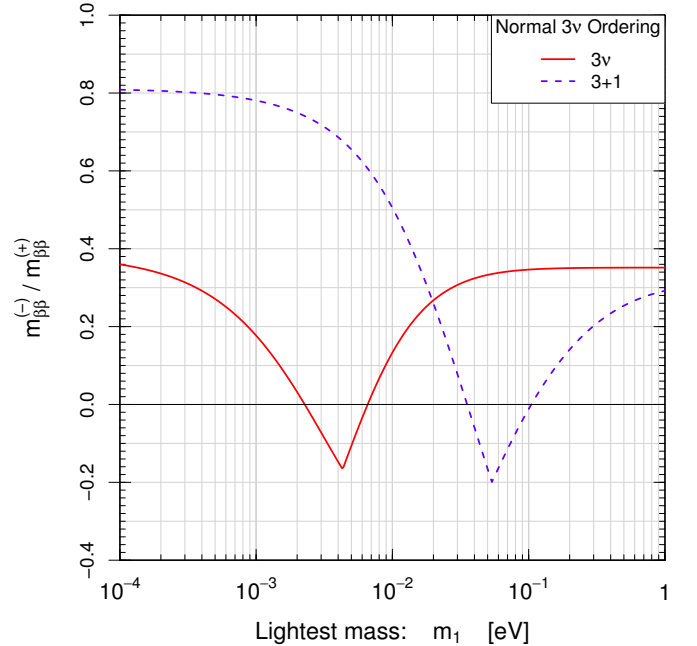


FIG. 2. Ratio $m_{\beta\beta}^{(-)}/m_{\beta\beta}^{(+)}$ (see Eq. (13)) as a function of m_1 for the best-fit values of the partial mass contributions in the case of 3ν and $3+1$ mixing with Normal Ordering of the three lightest neutrinos.

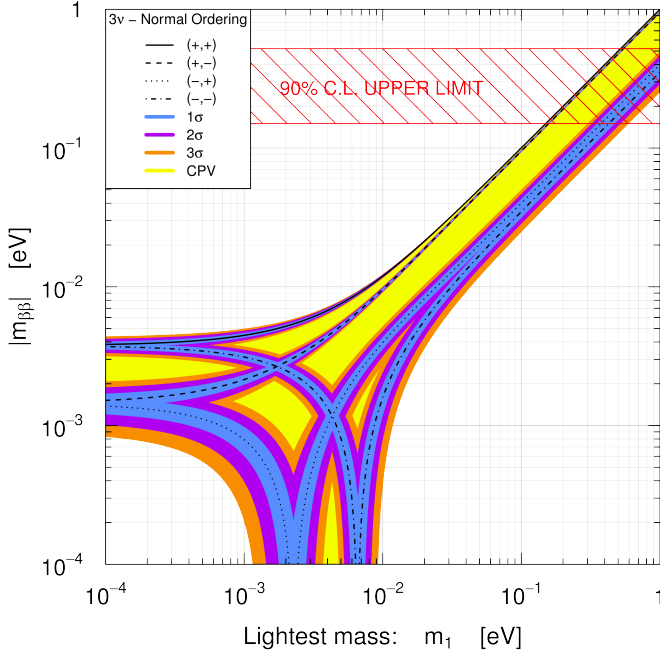


FIG. 3. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass m_1 in the case of 3ν mixing with Normal Ordering. The signs in the legend indicate the signs of $e^{i\alpha_2}, e^{i\alpha_3} = \pm 1$ for the four possible cases in which CP is conserved. The intermediate yellow region is allowed only in the case of CP violation. The 90% upper limit is explained in the main text.

neutrino mass m_{\min} :

$$m_1 = m_{\min}, \quad (8)$$

$$m_2 = \sqrt{m_{\min}^2 + \Delta m_{\text{SOL}}^2}, \quad (9)$$

$$m_3 = \sqrt{m_{\min}^2 + \Delta m_{\text{ATM}}^2 + \Delta m_{\text{SOL}}^2/2}. \quad (10)$$

Figure 1 shows the best-fit values and the 1σ , 2σ and 3σ allowed intervals of the three partial mass contributions to $|m_{\beta\beta}|$ in Eq. (1) as functions of the lightest mass m_1 . We calculated the confidence intervals using the χ^2 function

$$\chi_{3\nu}^2 = \chi^2(\Delta m_{\text{SOL}}^2) + \chi^2(\Delta m_{\text{ATM}}^2) + \chi^2(\sin^2 \vartheta_{12}) + \chi^2(\sin^2 \vartheta_{13}), \quad (11)$$

TABLE I. Ranges of m_1 , m_β and Σ for which there can be a complete cancellation of the three partial mass contributions to $|m_{\beta\beta}|$ for the best-fit values (b.f.) of the oscillation parameters and at 1σ , 2σ and 3σ in the case of 3ν mixing with Normal Ordering.

	b.f.	1σ	2σ	3σ
$m_1 [10^{-3} \text{ eV}]$	2.3 – 6.6	1.9 – 7.2	1.6 – 8.0	1.3 – 9.0
$m_\beta [10^{-2} \text{ eV}]$	0.9 – 1.1	0.9 – 1.2	0.8 – 1.3	0.8 – 1.4
$\Sigma [10^{-2} \text{ eV}]$	6.1 – 6.8	5.9 – 7.0	5.8 – 7.2	5.7 – 7.4

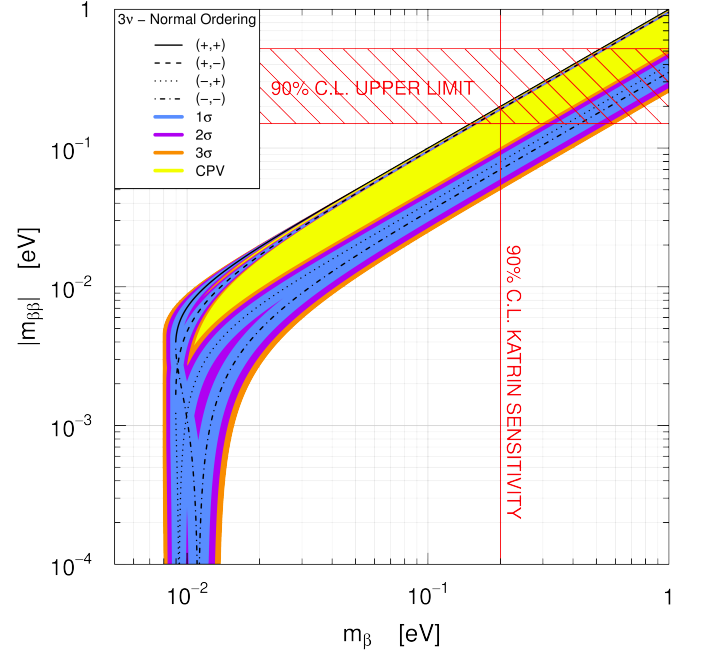


FIG. 4. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of effective electron neutrino mass m_β in Eq. (22) in the case of 3ν mixing with Normal Ordering. The legend is explained in the caption of Fig. 3. The limits are explained in the main text.

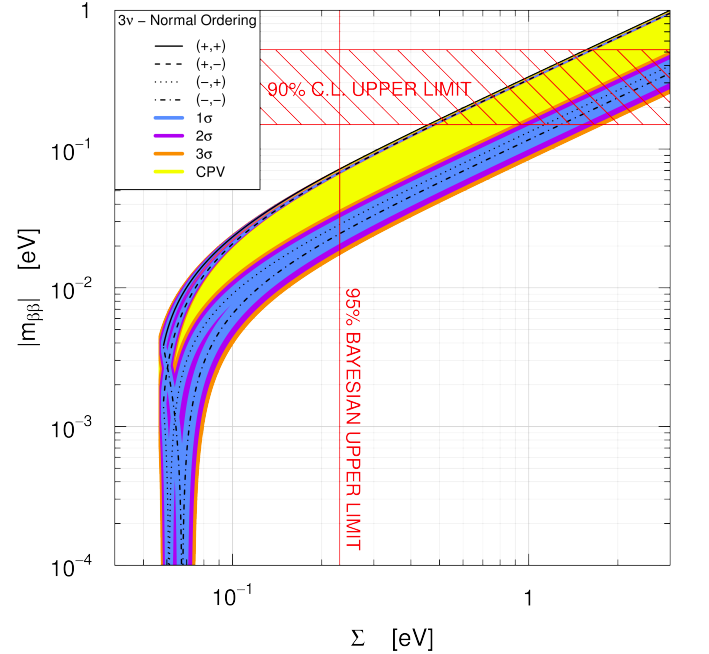


FIG. 5. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of sum of the neutrino masses Σ in Eq. (23) in the case of 3ν mixing with Normal Ordering. The legend is explained in the caption of Fig. 3. The limits are explained in the main text.

with the partial χ^2 's extracted from Fig. 3 of Ref. [3],

neglecting possible small correlations of the four mixing parameters¹. For each value of m_1 we calculated the confidence intervals for one degree of freedom. Although this method is in principle better than the method based on the propagation of errors of the parameters used by many authors, in practice it leads to similar results, because the χ^2 's of Δm_{SOL}^2 , Δm_{ATM}^2 , $\sin^2 \vartheta_{12}$ and $\sin^2 \vartheta_{13}$ in Fig. 3 of Ref. [3] are very well approximated by quadratic functions, which correspond to Gaussian uncertainties for which the method of propagation of errors is valid.

From Fig. 1 one can see that for $m_1 \lesssim 2 \times 10^{-3}$ eV the contribution of μ_2 is dominant and cannot be canceled by the smaller contributions of μ_1 and μ_3 for any values of the relative phase differences. In the interval $m_1 \approx (2 - 7) \times 10^{-3}$ eV cancellations are possible, mainly between μ_1 and μ_2 which have similar values, with the smaller contribution of μ_3 , which is about 2.3 times smaller than μ_2 . For $m_1 \gtrsim 7 \times 10^{-3}$ eV again there cannot be a complete cancellation, because the contribution of μ_1 is dominant.

The result for $|m_{\beta\beta}|$ is shown in Fig. 3, where we have plotted separately the allowed bands for the four possible cases in which CP is conserved ($\alpha_2, \alpha_3 = 0, \pi$) and the coefficients of the contributions are real. These are the extreme cases which determine the minimum and maximum values of $|m_{\beta\beta}|$. The areas between the CP-conserving curves correspond to values of $|m_{\beta\beta}|$ which are allowed only in the case of CP violation [31, 50–54] (although there is no manifest CP violation [55]).

Figure 3 shows also the 90% C.L. upper limit band for $|m_{\beta\beta}|$ estimated in Ref. [22] from the results of the KamLAND-Zen experiment [56] taking into account the uncertainties of the nuclear matrix element calculations. The reliability of this upper limit is supported by the upper limits with the same order of magnitude following from the results of the Heidelberg-Moscow [57], IGEX [58], GERDA [59], NEMO-3 [60], CUORICINO [61], and EXO [62] experiments.

From Fig. 3 one can see that, in agreement with the discussion above, there can be a complete cancellation of the three partial mass contributions to $|m_{\beta\beta}|$ for m_1 in the intervals given in Tab. I at different confidence levels.

The exact determination of the region in which there can be a complete cancellation of the partial mass contributions to $|m_{\beta\beta}|$ in the general case of N -neutrino mixing can be done in the following relatively simple way². For each value of the lightest mass m_1 let us denote by a the index of the largest mass contribution, i.e.

$$\mu_a \geq \mu_k \quad \text{for } k \neq a. \quad (12)$$

Then, we can consider the quantities

$$m_{\beta\beta}^{(\pm)} = \mu_a \pm \sum_{k \neq a} \mu_k. \quad (13)$$

The quantity $m_{\beta\beta}^{(+)}$ is always positive and represents the most favorable case, in which all the mass contribution add with the same phase, giving the maximum value of $|m_{\beta\beta}|$ for any value of the unknown phases:

$$|m_{\beta\beta}|_{\text{max}} = m_{\beta\beta}^{(+)}. \quad (14)$$

The quantity $m_{\beta\beta}^{(-)}$ represents the extreme case in which the phases of all the other partial mass contributions are equal and opposite to the phase of the largest mass contribution. It is evident that if $m_{\beta\beta}^{(-)} > 0$, the value of $m_{\beta\beta}$ gives the minimum possible value of $|m_{\beta\beta}|$ for any value of the unknown phases, because it corresponds to the maximal cancellation between μ_a and the maximum $\sum_{k \neq a} \mu_k$ of the other partial mass contributions. On the other hand, if

$$m_{\beta\beta}^{(-)} \leq 0, \quad (15)$$

there is an intermediate value of the phases which gives $|m_{\beta\beta}| = 0$. This can be seen clearly by writing $|m_{\beta\beta}|$ as

$$|m_{\beta\beta}| = \left| \mu_a + e^{i\alpha'} \mu' \right|, \quad (16)$$

with

$$\mu' = \left| \sum_{k \neq a} e^{i\xi_k} \mu_k \right|, \quad (17)$$

with an unknown phase α' and $N-2$ unknown phases ξ_k , of which one can be fixed to zero. The only possibility to have $|m_{\beta\beta}| = 0$ can be realized for $\alpha' = \pi$ if $\mu' = \mu_a$. This equality can occur only if

$$\mu'_{\text{min}} \leq \mu_a \leq \mu'_{\text{max}}, \quad (18)$$

where μ'_{min} and μ'_{max} are the minimum and maximum values of μ' for any value of the unknown phases ξ_k . Since we have always $\mu'_{\text{min}} \leq \mu_a$ because of Eq. (12) and

$$\mu'_{\text{max}} = \sum_{k \neq a} \mu_k, \quad (19)$$

the inequality in Eq. (15) is the necessary and sufficient condition for having $|m_{\beta\beta}| = 0$ for some value of the unknown phases. Hence, the minimum value of $|m_{\beta\beta}|$ is given by

$$|m_{\beta\beta}|_{\text{min}} = \max \left[m_{\beta\beta}^{(-)}, 0 \right], \quad (20)$$

which can also be written as [63]³

$$|m_{\beta\beta}|_{\text{min}} = \max[2\mu_k - |m_{\beta\beta}|_{\text{max}}, 0]. \quad (21)$$

¹ The only significant correlations discussed in Ref. [3] are those which involve the mixing angle ϑ_{23} , which is irrelevant for neutrinoless double- β decay, and the Dirac phase, whose effect in neutrinoless double- β decay is masked by the two unknown Majorana phases.

² Other ways are discussed in Refs. [30, 63, 64].

³ Equation (21) is the generalization to N -neutrino mixing of that obtained with a different proof in Ref. [63] in the case of 3ν mixing.

Fig. 2 shows the value of the ratio $m_{\beta\beta}^{(-)}/m_{\beta\beta}^{(+)}$ as a function of m_1 for the best-fit values of the partial mass contributions, which is sufficient for the determination of the interval of m_1 for which there can be a complete cancellation of the partial mass contributions. One can see that in the case of 3ν mixing $m_{\beta\beta}^{(-)}$ is negative and it is possible that $|m_{\beta\beta}| = 0$ only in the interval of m_1 given in Tab. I.

Let us now consider the opposite possibility that $|m_{\beta\beta}|$ is larger than about 0.01 eV, which is a value that may be explored experimentally in the near future. From Fig. 3 one can see that $|m_{\beta\beta}| \gtrsim 0.01$ eV can be realized only for $m_1 \gtrsim 0.008$ eV. This range of m_1 corresponds to almost degenerate m_1 and m_2 , because $\sqrt{\Delta m_{\text{SOL}}^2} \approx 8.7 \times 10^{-3}$ eV. Hence, it will be very difficult to measure $|m_{\beta\beta}|$ if there is a normal hierarchy of neutrino masses ($m_1 \ll m_2 \ll m_3$) for any value of the unknown phases α_2 and α_3 in Eq. (1).

One can also see from Fig. 3 that $|m_{\beta\beta}| \gtrsim 0.01$ eV is realized independently of the values of the unknown phases α_2 and α_3 for $m_1 \gtrsim 0.04$ eV, which is close to the region $m_1 \gtrsim \sqrt{\Delta m_{\text{ATM}}^2} \approx 0.05$ eV in which all the three neutrino masses are quasidegenerate.

Figure 3 gives a clear view of the possible values of $|m_{\beta\beta}|$ depending on the scale of the lightest mass m_1 , but it is of little practical usefulness, because it will be very difficult to measure directly the value of m_1 . In practice, the investigation of the absolute values of neutrino masses is performed through the measurements of the effective electron neutrino mass

$$m_\beta = \sqrt{|U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2} \quad (22)$$

in β -decay experiments [23, 24] and through the measurement of the sum of the neutrino masses

$$\Sigma = m_1 + m_2 + m_3 \quad (23)$$

in cosmological experiments (see, for example, Ref. [65]). Hence, it is useful to calculate the allowed regions in the m_β - $|m_{\beta\beta}|$ and Σ - $|m_{\beta\beta}|$ planes [66–69], which are shown in Figs. 4 and 5. In this case, the confidence intervals are calculated using the χ^2 function in Eq. (11) with two degrees of freedom. We have plotted separately the allowed bands for the four possible cases in which CP is conserved ($\alpha_2, \alpha_3 = 0, \pi$), in order to show the regions in which CP is violated. Potentially the possibility of measuring values of $|m_{\beta\beta}|$ and m_β or Σ in these regions is very exciting for the discovery of CP violation generated by the Majorana phases⁴, but in practice such measurement is very difficult because it would require a precision which seems

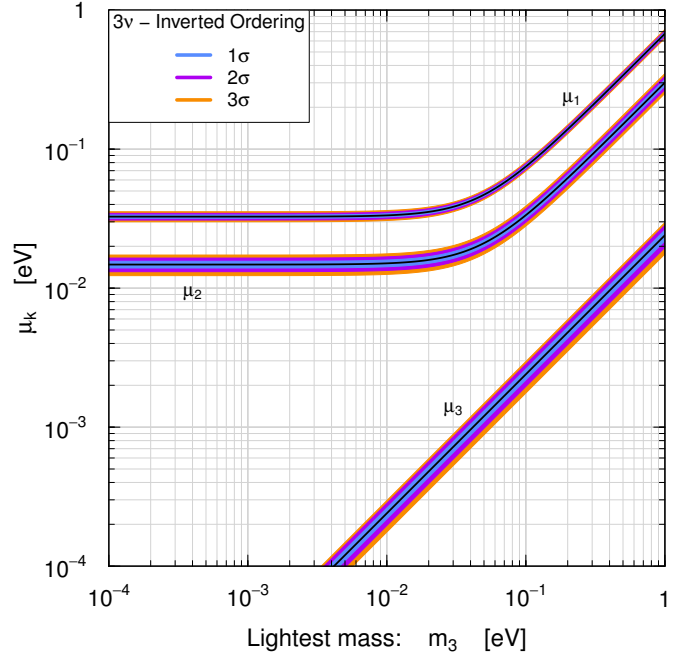


FIG. 6. Best-fit values (b.f.) and 1σ , 2σ and 3σ allowed intervals of the three partial mass contributions to $|m_{\beta\beta}|$ in Eq. (1) as functions of the lightest mass m_3 in the case of 3ν mixing with Inverted Ordering.

to be beyond what can be currently envisioned [71–74], especially taking into account the current uncertainty of the calculation of the nuclear matrix element in neutrinoless double- β decay (see Refs. [22, 75–77]).

Figures 4 and 5 show the same 90% C.L. upper limit band for $|m_{\beta\beta}|$ as in Fig. 3. In addition, Fig. 4 shows⁵ the 90% C.L. sensitivity on m_β of the KATRIN experiment [78], which is scheduled to start data taking in 2016, and Fig. 5 shows the 95% bayesian upper limit on Σ obtained by the Planck collaboration [65].

The intervals of m_β and Σ for which there can be a complete cancellation of the three partial mass contributions to $|m_{\beta\beta}|$ are given in Tab. I. On the other hand, from Figs. 4 and 5 one can see that $|m_{\beta\beta}| \gtrsim 0.01$ eV for any value of the unknown phases α_2 and α_3 for $m_\beta \gtrsim 0.05$ eV and $\Sigma \gtrsim 0.15$ eV. The 3σ lower bounds for m_β and Σ are, respectively, 0.8×10^{-2} eV and 5.6×10^{-2} eV.

⁴ The phases α_2 and α_3 in Eq. (1) depend on the values of one Dirac phase and two Majorana phases in the neutrino mixing matrix (see Ref. [70]). The Dirac phase can be measured in neutrino oscillation experiments and there is some indication on its value [3–5]. On the other hand, the values of the two Majorana phases

can be measured only in lepton-number violating processes such as neutrinoless double- β decay. In the future, if the Dirac phase will be measured, CP violation in neutrinoless double- β decay may provide information on the Majorana phases.

⁵ The most stringent current upper limits on m_β obtained in the Mainz ($m_\beta < 2.3$ eV at 95% C.L.) [23] and Troitsk ($m_\beta < 2.1$ eV 95% C.L.) [24] experiments are out of the scale in Fig. 4.

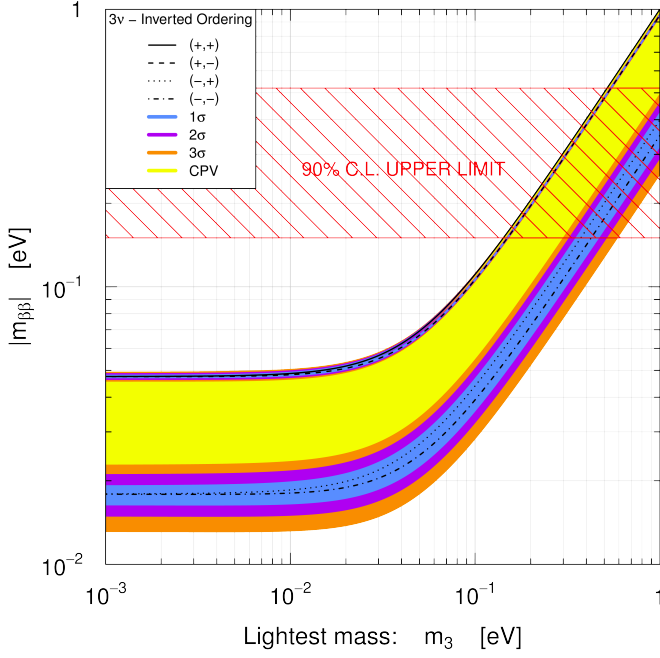


FIG. 7. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of lightest neutrino mass in the three neutrino case for the Inverted Ordering. The legend is explained in the caption of Fig. 3. The 90% upper limit is explained in Section II A.

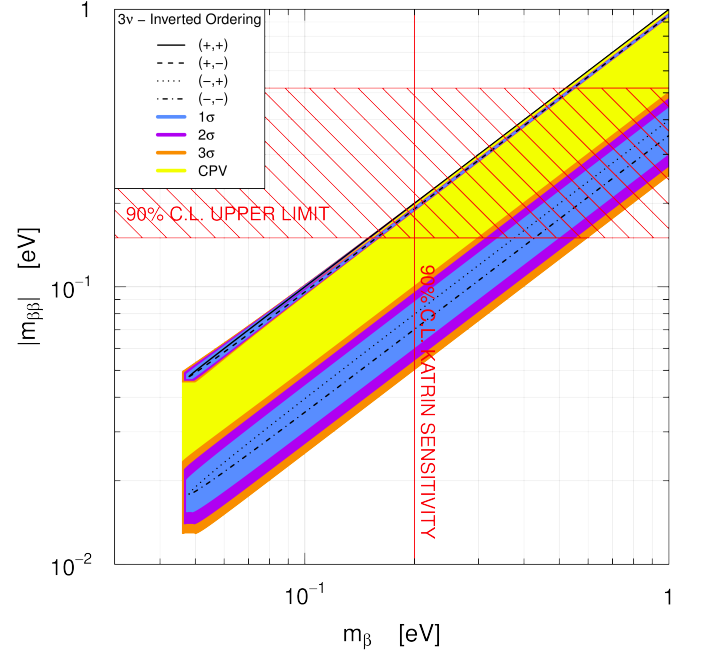


FIG. 9. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of effective electron neutrino mass m_{β} in Eq. (22) in the case of 3ν mixing with Inverted Ordering. The legend is explained in the caption of Fig. 3. The limits are explained in Section II A.

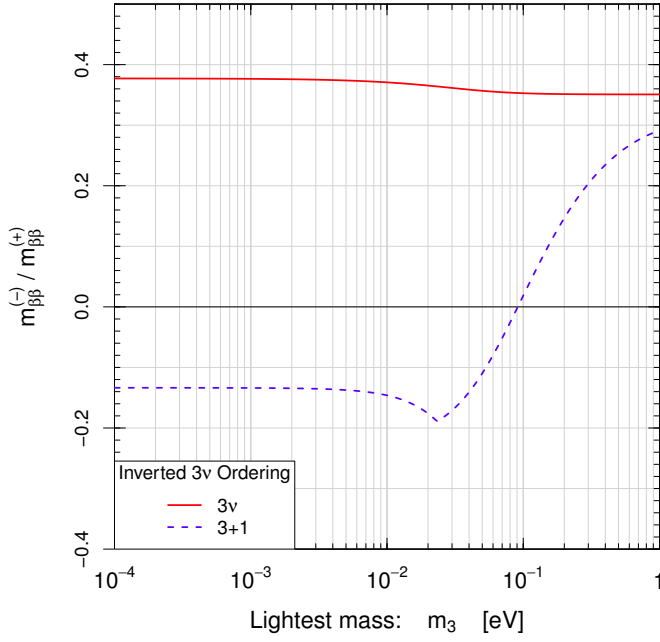


FIG. 8. Ratio $m_{\beta\beta}^{(-)}/m_{\beta\beta}^{(+)}$ (see Eq. (13)) as a function of m_3 for the best-fit values of the partial mass contributions in the case of 3ν and $3+1$ mixing with Inverted Ordering of the three lightest neutrinos.

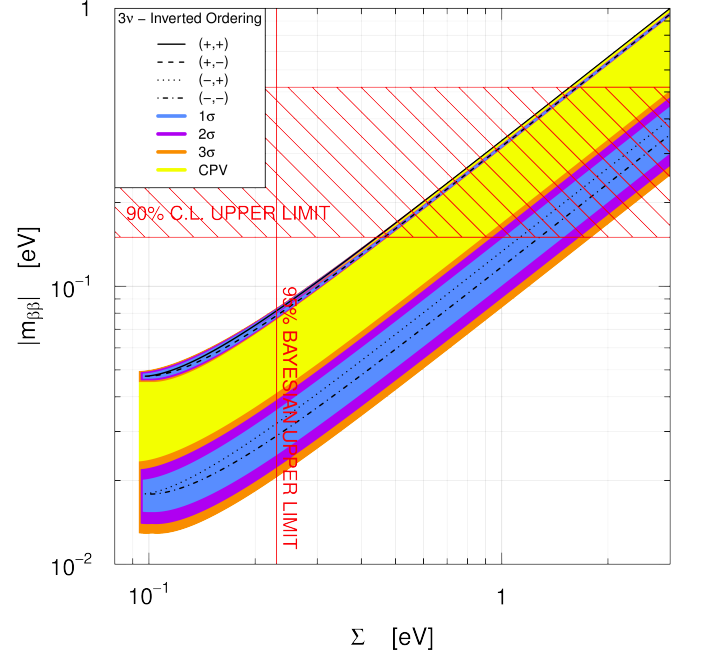


FIG. 10. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of sum of the neutrino masses Σ in Eq. (23) in the case of 3ν mixing with Inverted Ordering. The legend is explained in the caption of Fig. 3. The limits are explained in Section II A.

B. Inverted Ordering

In the case of Inverted Ordering (IO), the expressions of the neutrino masses in terms of the lightest neutrino mass m_{\min} are:

$$m_3 = m_{\min}, \quad (24)$$

$$m_1 = \sqrt{m_{\min}^2 + \Delta m_{\text{ATM}}^2 - \Delta m_{\text{SOL}}^2/2}, \quad (25)$$

$$m_2 = \sqrt{m_{\min}^2 + \Delta m_{\text{ATM}}^2 + \Delta m_{\text{SOL}}^2/2}. \quad (26)$$

Figure 6 shows the best-fit values and the 1σ , 2σ and 3σ allowed intervals of the three partial mass contributions to $|m_{\beta\beta}|$ in Eq. (1) as functions of the lightest mass m_3 . One can see that μ_1 is always dominant, because ϑ_{12} is smaller than $\pi/4$ and $|U_{e1}| > |U_{e2}| > |U_{e3}|$. Therefore, in the case of Inverted Ordering there cannot be a complete cancellation of the three mass contributions to $|m_{\beta\beta}|$ (Fig. 8 shows that $m_{\beta\beta}^{(-)}$ is always positive) and we obtain from Fig. 7 the lower bounds

$$|m_{\beta\beta}| > 1.6 (1\sigma), 1.5 (2\sigma), 1.3 (3\sigma) \times 10^{-2} \text{ eV}. \quad (27)$$

In the case of an Inverted Hierarchy ($m_3 \ll m_1 < m_2$) we also have the upper bounds

$$|m_{\beta\beta}| < 4.8 (1\sigma), 4.9 (2\sigma), 4.9 (3\sigma) \times 10^{-2} \text{ eV}. \quad (28)$$

The next generations of neutrinoless double-beta decay experiments (see Refs. [36–41]) will try to explore the range of $|m_{\beta\beta}|$ between the limits in Eqs. (27) and (28), testing the Majorana nature of neutrinos in the case of an Inverted Hierarchy.

Figures 9 and 10 show the correlation between $|m_{\beta\beta}|$ and the measurable quantities m_β and Σ in the Inverted Ordering. Since in this case both m_β and Σ have relatively large lower bounds ($4.6 \times 10^{-2} \text{ eV}$ and $9.4 \times 10^{-2} \text{ eV}$, respectively, at 3σ) there is a concrete possibility that near-future experiments will determine an allowed region in these plots if in nature there are only three neutrinos with Inverted Ordering.

III. 3+1 MIXING

In this section we consider the case of 3+1 mixing in which there is a new massive neutrino ν_4 at the eV scale which is mainly sterile. As explained in Section I, 3+1 mixing is motivated [18, 19] by the explanation of the reactor, Gallium and LSND anomalies, which requires the existence of a new squared-mass difference $\Delta m_{\text{SBL}}^2 \sim 1 \text{ eV}^2$. In this case, the effective Majorana mass in neutrinoless double-beta decay is given by

$$|m_{\beta\beta}| = |\mu_1 + \mu_2 e^{i\alpha_2} + \mu_3 e^{i\alpha_3} + \mu_4 e^{i\alpha_4}|, \quad (29)$$

with the partial mass contributions given by Eq. (2). The contribution of ν_4 enters with a totally unknown new

phase α_4 that must be varied from 0 to 2π as α_2 and α_3 in order to calculate the predictions of the value of $|m_{\beta\beta}|$.

The absolute values of the relevant first row of the 4×4 mixing matrix U is given by the simple extension of the standard parameterization:

$$|U_{e1}| = \cos \vartheta_{14} \cos \vartheta_{13} \cos \vartheta_{12}, \quad (30)$$

$$|U_{e2}| = \cos \vartheta_{14} \cos \vartheta_{13} \sin \vartheta_{12}, \quad (31)$$

$$|U_{e3}| = \cos \vartheta_{14} \sin \vartheta_{13}, \quad (32)$$

$$|U_{e4}| = \sin \vartheta_{14}. \quad (33)$$

Since in the case of 3+1 neutrino mixing, as well as in any extension of the standard 3ν mixing, the ordering of the three standard massive neutrinos is not known, in the following two subsections we consider separately the two cases of Normal and Inverted Ordering of ν_1, ν_2, ν_3 . The values of their masses as functions of the lightest mass m_{\min} are given by Eqs. (8)–(10) in the Normal Ordering and by Eqs. (24)–(26) in the Inverted Ordering. However, in both cases we have

$$m_4 \simeq \sqrt{m_{\min}^2 + \Delta m_{\text{SBL}}^2}, \quad (34)$$

neglecting the contributions of Δm_{SOL}^2 and Δm_{ATM}^2 , which are much smaller than Δm_{SBL}^2 . We calculated the confidence intervals using the χ^2 function

$$\chi_{3+1}^2 = \chi_{3\nu}^2 + \chi^2(\Delta m_{\text{SBL}}^2, \sin^2 \vartheta_{14}), \quad (35)$$

with $\chi_{3\nu}^2$ defined in Eq. (11) and $\chi^2(\Delta m_{\text{SBL}}^2, \sin^2 \vartheta_{14})$ obtained from an update [34, 35] of the global fit of short-baseline neutrino oscillation data presented in Ref. [19].

After Eq. (11) we noted that in the case of 3ν mixing our statistical method for the calculation of the uncertainty of $|m_{\beta\beta}|$ and the usual method based on the propagation of errors lead to similar results, because the χ^2 's of the relevant 3ν mixing parameters are very well approximated by quadratic functions. On the other hand, the usual propagation of errors is inaccurate in the case of 3+1 mixing, because the marginal χ^2 's of Δm_{SBL}^2 and $\sin^2 \vartheta_{14}$ are not quadratic. Moreover, there are significant correlations between Δm_{SBL}^2 and $\sin^2 \vartheta_{14}$ (see Fig. 3 of Ref. [19]) which are taken into account in $\chi^2(\Delta m_{\text{SBL}}^2, \sin^2 \vartheta_{14})$.

TABLE II. Ranges of m_1 , m_β and Σ for which there can be a complete cancellation of the four partial mass contributions to $|m_{\beta\beta}|$ for the best-fit values (b.f.) of the oscillation parameters and at 1σ , 2σ and 3σ in the case of 3+1 mixing with Normal Ordering of the three lightest neutrinos.

	b.f.	1σ	2σ	3σ
$m_1 [10^{-2} \text{ eV}]$	3.5 – 10.5	2.8 – 12.5	2.0 – 16.3	1.3 – 20.0
$m_\beta [10^{-2} \text{ eV}]$	3.6 – 10.5	2.6 – 14.3	1.9 – 17.8	1.5 – 22.7
$\Sigma [10^{-2} \text{ eV}]$	13.3 – 32.6	10.4 – 43.8	8.7 – 53.9	7.6 – 68.5

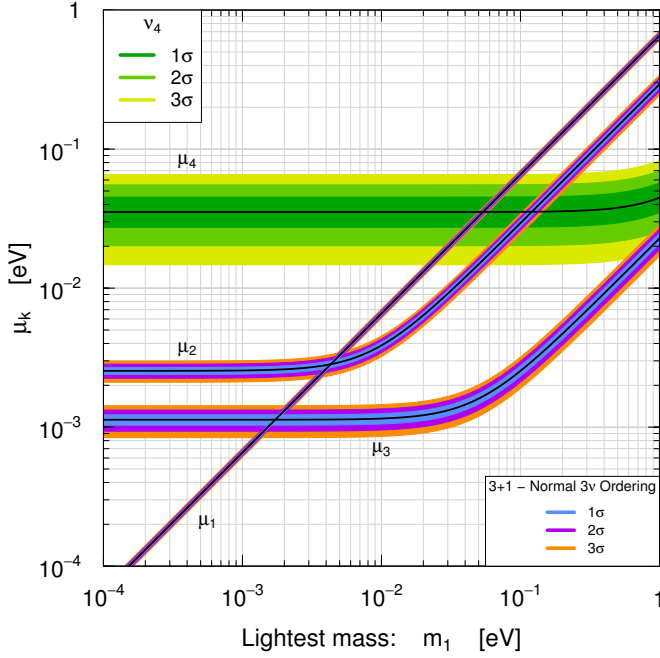


FIG. 11. Best-fit values (b.f.) and 1σ , 2σ and 3σ allowed intervals of the four partial mass contributions to $|m_{\beta\beta}|$ in Eq. (29) as functions of the lightest mass m_1 in the case of 3+1 mixing with Normal Ordering of the three lightest neutrinos.

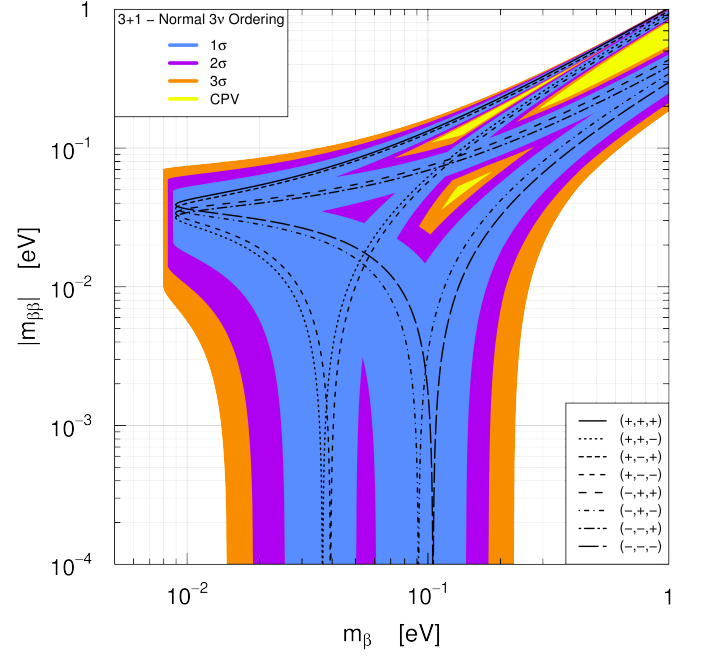


FIG. 13. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of effective electron neutrino mass m_β in Eq. (22) in the case of 3+1 mixing with Normal Ordering of the three lightest neutrinos. The legend is explained in the caption of Fig. 12.

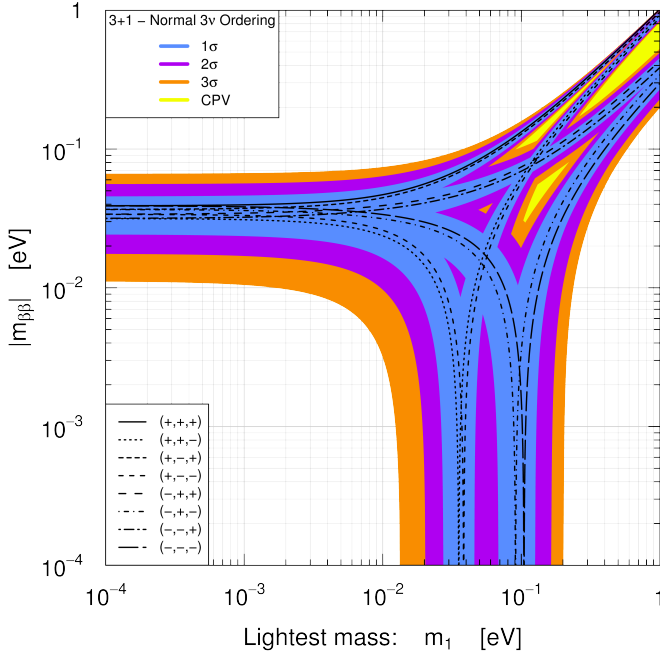


FIG. 12. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass m_1 in the case of 3+1 mixing with Normal Ordering of the three lightest neutrinos. The signs in the legend indicate the signs of $e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4} = \pm 1$ for the four possible cases in which CP is conserved. The intermediate yellow region is allowed only in the case of CP violation.

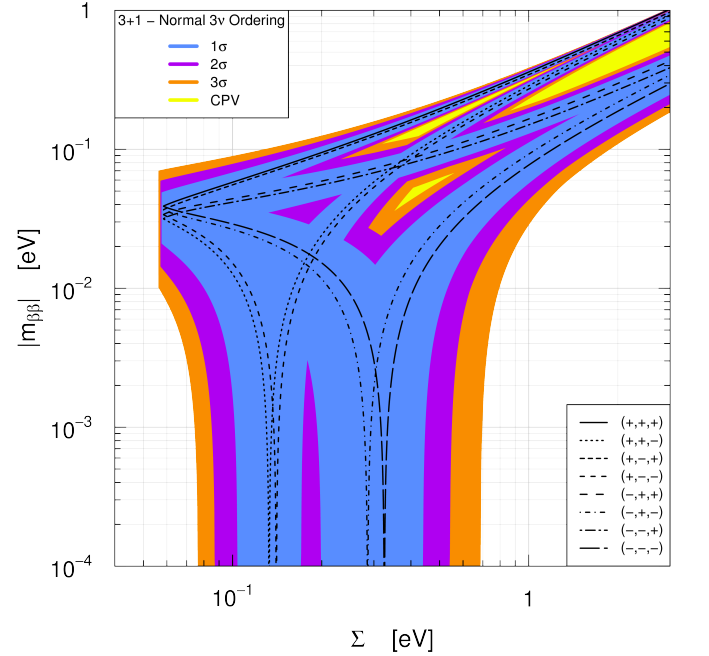


FIG. 14. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of sum of the three lightest neutrino masses Σ in Eq. (23) in the case of 3+1 mixing with Normal Ordering of the three lightest neutrinos. The legend is explained in the caption of Fig. 12.

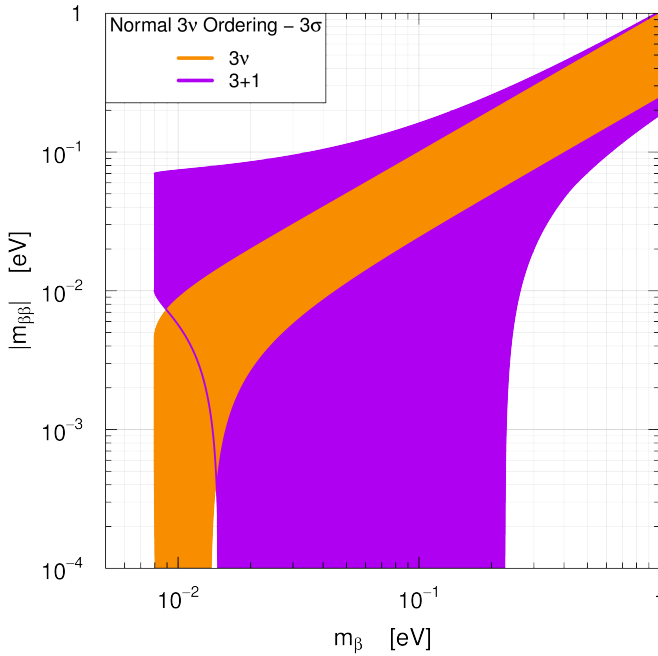


FIG. 15. Comparison of the 3σ allowed regions in the m_β – $|m_{\beta\beta}|$ plane in the cases of 3ν and $3+1$ mixing with Normal Ordering of the three lightest neutrinos.

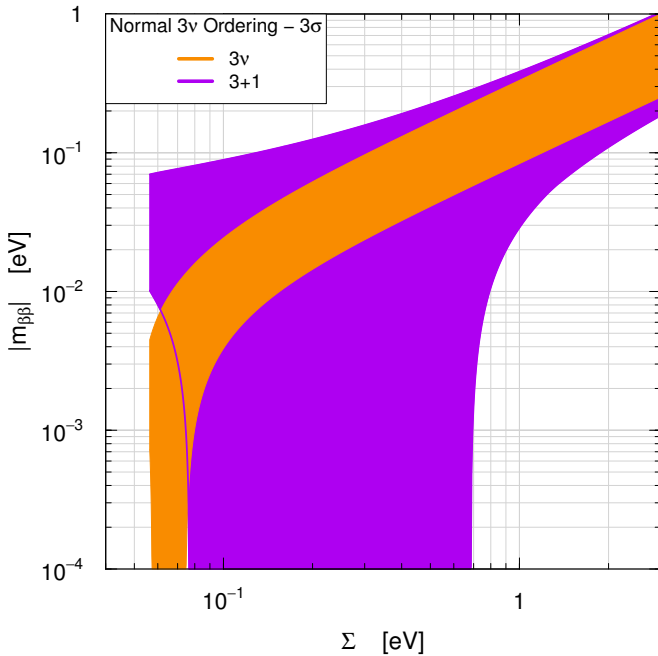


FIG. 16. Comparison of the 3σ allowed regions in the Σ – $|m_{\beta\beta}|$ plane in the cases of 3ν and $3+1$ mixing with Normal Ordering of the three lightest neutrinos.

A. Normal Ordering

Figure 11 shows a comparison of the best-fit value and the 1σ , 2σ and 3σ allowed intervals of the partial contribution μ_4 as a function of the lightest mass m_1 with

those of μ_1, μ_2, μ_3 , which are slightly different from those in Fig. 1 because of the contribution of ν_{14} in Eqs. (30)–(32). One can see that in the $3+1$ case it is not possible to get a total cancellation of $|m_{\beta\beta}|$ in the interval $m_1 \approx (2 - 7) \times 10^{-3}$ eV as in the case of 3ν mixing (see Tab. I), because in this interval of m_1 the contribution of μ_4 is dominant. However, there is a range of higher values of m_1 between about 0.02 and 0.2 eV in which μ_4 and μ_1 have similar values, leading to a possible total cancellation. For $m_1 \gtrsim 0.2$ eV a total cancellation is again not possible because the contribution of μ_1 is dominant. This behavior is confirmed by Fig. 2, where one can see that the value of $m_{\beta\beta}^{(-)}$ (see Eq. (13)) corresponding to the best-fit values of the partial mass contributions is negative for $0.035 \lesssim m_1 \lesssim 0.1$ eV.

Figure 12 shows the allowed values of $|m_{\beta\beta}|$ as a function of m_1 at different confidence levels. The corresponding intervals of m_1 for which there can be a total cancellation of $|m_{\beta\beta}|$ are given in Tab. II.

In Fig. 12 we have plotted separately the allowed bands for the eight possible cases in which CP is conserved ($\alpha_2, \alpha_3, \alpha_4 = 0, \pi$), that are the extreme cases which determine the minimum and maximum values of $|m_{\beta\beta}|$. The areas between the CP-conserving allowed bands correspond to values of $|m_{\beta\beta}|$ which are allowed only in the case of CP violation. Unfortunately, these areas are visibly smaller than those in Fig. 3 in the case of 3ν mixing. This is due to the relatively large uncertainty of μ_4 , which can be seen clearly in Fig. 11. This uncertainty broadens the allowed bands corresponding to the CP-conserving cases, leaving little intermediate space. In any case, even if the uncertainty of μ_4 will be reduced in the future, there cannot be a region which is allowed only in the case of CP-violation for $m_1 \lesssim 10^{-2}$ eV, where μ_4 is dominant and the CP-violating phases are irrelevant. In fact, all the best-fit CP-conserving curves have approximately the same value for $m_1 \lesssim 10^{-2}$ eV.

As in the case of 3ν mixing, the plot in Fig. 12 of $|m_{\beta\beta}|$ as a function of the lightest mass m_1 is useful because it gives a clear view of the different possibilities for the value of $|m_{\beta\beta}|$, but in practice it will be very difficult to determine experimentally an allowed region in this plot because of the difficulty of measuring the value of the lightest mass. Therefore, we calculated also the allowed regions in the m_β – $|m_{\beta\beta}|$ and Σ – $|m_{\beta\beta}|$ planes shown in Figs. 13 and 14, with the quantities m_β and Σ defined in Eqs. (22) and (23) as in the case of 3ν mixing in terms of the three standard neutrino masses only. The reason of this choice is that m_β and Σ are measurable quantities also in the $3+1$ scheme. Indeed, considering β decay, m_β quantifies approximately the deviation of the end-point of the electron spectrum due to neutrino masses smaller than the experimental energy resolution [79–83], whereas the effect of the larger mass m_4 is a kink of the Kurie function (see Ref. [70]). In cosmology, the effects of the larger mass m_4 can be disentangled from those of the smaller masses, because ν_4 becomes non-relativistic shortly after matter-radiation equality, much earlier than

ν_1, ν_2, ν_3 . Moreover, it is possible that the contribution of m_4 to the energy density of the Universe is suppressed, for example by a large lepton asymmetry [84–88], or an enhanced background potential due to new interactions in the sterile sector [89–95], or a larger cosmic expansion rate at the time of sterile neutrino production [96], or MeV dark matter annihilation [97].

From Figs. 13 and 14 one can see that the intervals of m_β and Σ for which there can be a complete cancellation of the three partial mass contributions to $|m_{\beta\beta}|$ (given in Tab. II) are much larger than those in Figs. 4 and 5 for the standard 3ν mixing case, and $|m_{\beta\beta}| \gtrsim 0.01$ eV for any value of the unknown phases $\alpha_2, \alpha_3, \alpha_4$ only for the relatively large values $m_\beta \gtrsim 0.25$ eV and $\Sigma \gtrsim 0.8$ eV.

It is useful to compare the allowed regions m_β – $|m_{\beta\beta}|$ and Σ – $|m_{\beta\beta}|$ planes obtained in the cases of 3ν and $3+1$ mixing with Normal Ordering of the three lightest neutrinos. Figures 15 and 16 show this comparison for the 3σ allowed regions. One can see that, if the Normal Ordering will be established by oscillation experiments (see Refs. [1, 2]), with measurements of m_β and $|m_{\beta\beta}|$ and/or Σ and $|m_{\beta\beta}|$ it may be possible to distinguish 3ν mixing and $3+1$ mixing if the measured values select a region which is allowed only in one of the two cases. It is interesting that there are two regions allowed only to $3+1$ mixing: one with $|m_{\beta\beta}|$ smaller than that in the case of 3ν mixing and one with $|m_{\beta\beta}|$ larger than that in the case of 3ν mixing. At least a part of the second region is accessible to the next generation of neutrinoless double-beta decay experiments (see Refs. [36–41]). The only β -decay experiment under preparation with the aim of exploring the sub-eV region of m_β is KATRIN [98], which will have a sensitivity of about 0.2 eV that is not sufficient to explore the upper part of the region in Fig. 15 allowed only in the case of $3+1$ mixing. On the other hand, cosmological observation may be able to measure the sum of the three light neutrino masses down to the lower limit of about 5.6×10^{-2} eV [99].

B. Inverted Ordering

The best-fit value and the 1σ , 2σ and 3σ allowed intervals of the partial mass contributions to $|m_{\beta\beta}|$ in the case of $3+1$ mixing with Inverted Ordering of the three lightest neutrinos are shown in Fig. 17. One can see that there

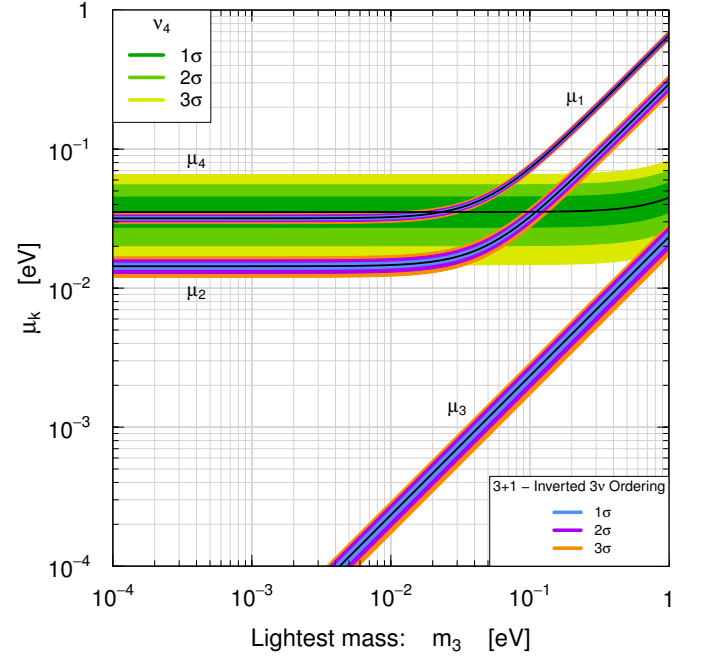


FIG. 17. Best-fit values (b.f.) and 1σ , 2σ and 3σ allowed intervals of the four partial mass contributions to $|m_{\beta\beta}|$ in Eq. (29) as functions of the lightest mass m_3 in the case of $3+1$ mixing with Inverted Ordering of the three lightest neutrinos.

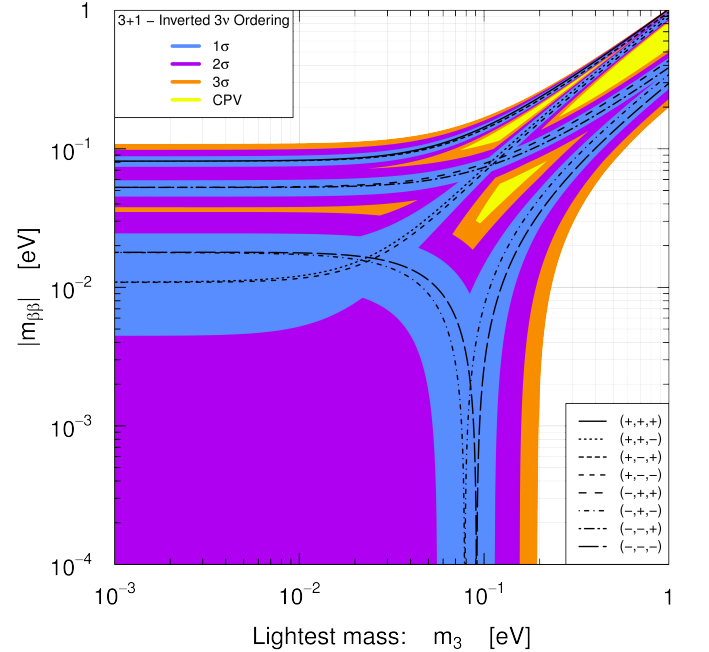


FIG. 18. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass m_3 in the case of $3+1$ mixing with Inverted Ordering of the three lightest neutrinos. The legend is explained in the caption of Fig. 12.

TABLE III. Ranges of m_3 , m_β and Σ for which there can be a complete cancellation of the four partial mass contributions to $|m_{\beta\beta}|$ for the best-fit values (b.f.) of the oscillation parameters and at 1σ , 2σ and 3σ in the case of $3+1$ mixing with Inverted Ordering of the three lightest neutrinos.

	b.f.	1σ	2σ	3σ
m_3 [10^{-2} eV]	< 9.1	< 11.4	< 15.5	< 19.3
m_β [10^{-2} eV]	4.8 – 10.3	4.8 – 14.2	4.7 – 17.6	4.6 – 22.5
Σ [10^{-2} eV]	9.8 – 29.9	9.7 – 41.8	9.5 – 52.3	9.4 – 67.1

can be a total cancellation between the partial contribution μ_4 and the dominant μ_1 in the case of 3ν mixing (see Fig. 6) for $m_3 \lesssim 0.1$ eV. Indeed, Fig. 8 shows that the

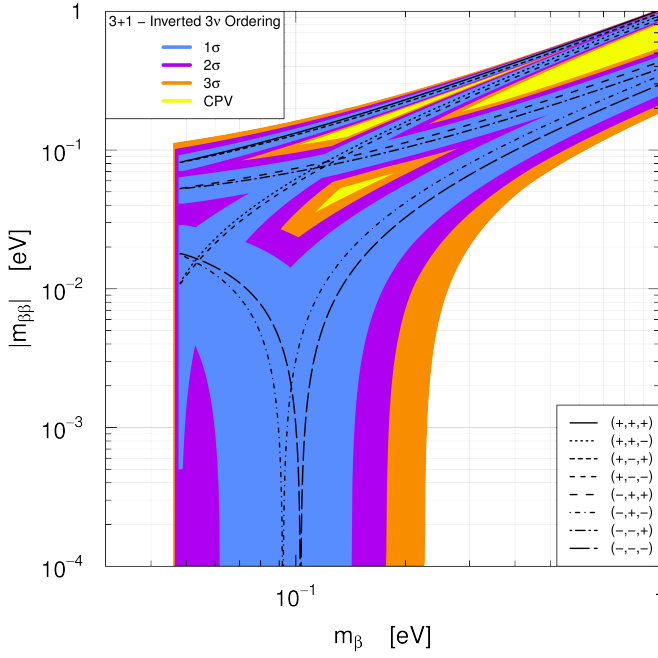


FIG. 19. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of sum of the three lightest neutrino masses Σ in Eq. (23) in the case of 3+1 mixing with Inverted Ordering of the three lightest neutrinos. The legend is explained in the caption of Fig. 12.

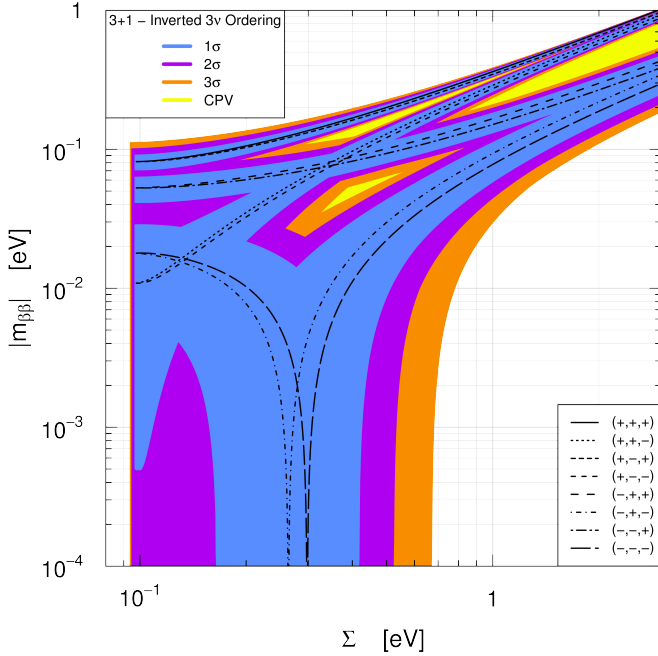


FIG. 20. Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of effective electron neutrino mass m_{β} in Eq. (22) in the case of 3+1 mixing with Inverted Ordering of the three lightest neutrinos. The legend is explained in the caption of Fig. 12.

value of $m_{\beta\beta}^{(-)}$ (see Eq. (13)) corresponding to the best-fit

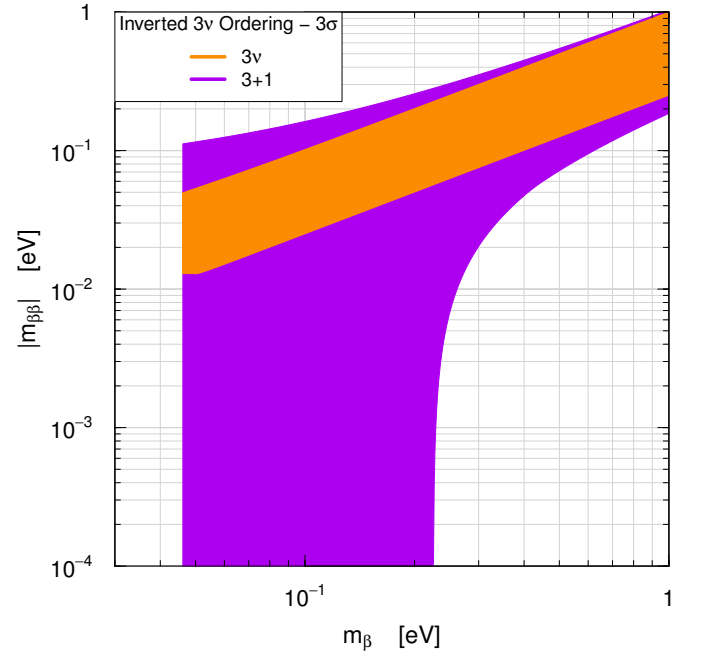


FIG. 21. Comparison of the 3σ allowed regions in the m_{β} – $|m_{\beta\beta}|$ plane in the cases of 3ν and $3+1$ mixing with Inverted Ordering of the three lightest neutrinos.

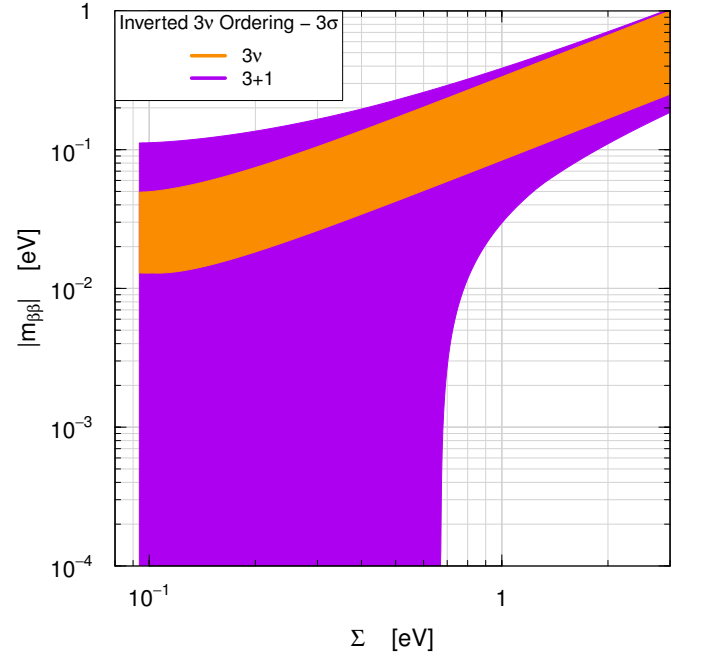


FIG. 22. Comparison of the 3σ allowed regions in the Σ – $|m_{\beta\beta}|$ plane in the cases of 3ν and $3+1$ mixing with Inverted Ordering of the three lightest neutrinos.

values of the partial mass contributions is negative for $m_3 \lesssim 0.09$ eV.

Figure 18 depicts the allowed regions in the m_3 – $|m_{\beta\beta}|$ plane. Comparing Fig. 18 with Fig. 7 one can see that the predictions for $|m_{\beta\beta}|$ are completely different in

the 3ν and $3+1$ cases if there is an Inverted Ordering of the three lightest neutrinos, in agreement with the discussions in Refs. [13, 25–33]. The ranges of values of m_3 for which there can be a complete cancellation of $|m_{\beta\beta}|$ are given in Tab. III.

Figures 19 and 20 show the allowed regions in the m_β – $|m_{\beta\beta}|$ and Σ – $|m_{\beta\beta}|$ planes. Figures 21 and 22 show the comparison of the 3σ allowed regions in the same planes in the cases of 3ν and $3+1$ mixing with Inverted Ordering of the three lightest neutrinos. If the Inverted Ordering will be established by oscillation experiments (see Refs. [1, 2]), it will be possible to exclude 3ν mixing in favor of $3+1$ by restricting m_β and $|m_{\beta\beta}|$ or Σ and $|m_{\beta\beta}|$ in the corresponding large region at small $|m_{\beta\beta}|$ allowed only in the $3+1$ case.

IV. CONCLUSIONS

We have presented accurate calculations of the effective Majorana mass $|m_{\beta\beta}|$ in neutrinoless double- β decay in the standard case of 3ν mixing and in the case of $3+1$ neutrino mixing indicated by the reactor, Gallium and LSND anomalies (see Refs. [18, 19]). We have taken into account the uncertainties of the standard 3ν mixing parameters obtained in the global fit of solar, atmospheric and long-baseline reactor and accelerator neutrino oscillation data presented in Ref. [3] and the uncertainties on the additional mixing parameters in the $3+1$ case obtained from an update [34, 35] of the global fit of short-

baseline neutrino oscillation data presented in Ref. [19].

We have shown that the predictions for $|m_{\beta\beta}|$ in the cases of 3ν and $3+1$ mixing are quite different, in agreement with the previous discussions in Refs. [13, 25–33]. Our paper improves these discussions by taking into account the uncertainties of all the mixing parameters and presenting all the results at 1σ , 2σ and 3σ .

We have presented accurate comparisons of the allowed regions in the planes m_β – $|m_{\beta\beta}|$ and Σ – $|m_{\beta\beta}|$ of measurable quantities, taking into account the two possibilities of Normal and Inverted Ordering of the three lightest neutrinos. We have shown that future measurements of these quantities may distinguish the 3ν and $3+1$ cases if the mass ordering is determined by oscillation experiments (see Refs. [1, 2]).

We have also introduced in Section II A a relatively simple method to determine the minimum value of $|m_{\beta\beta}|$ in the general case of N -neutrino mixing.

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